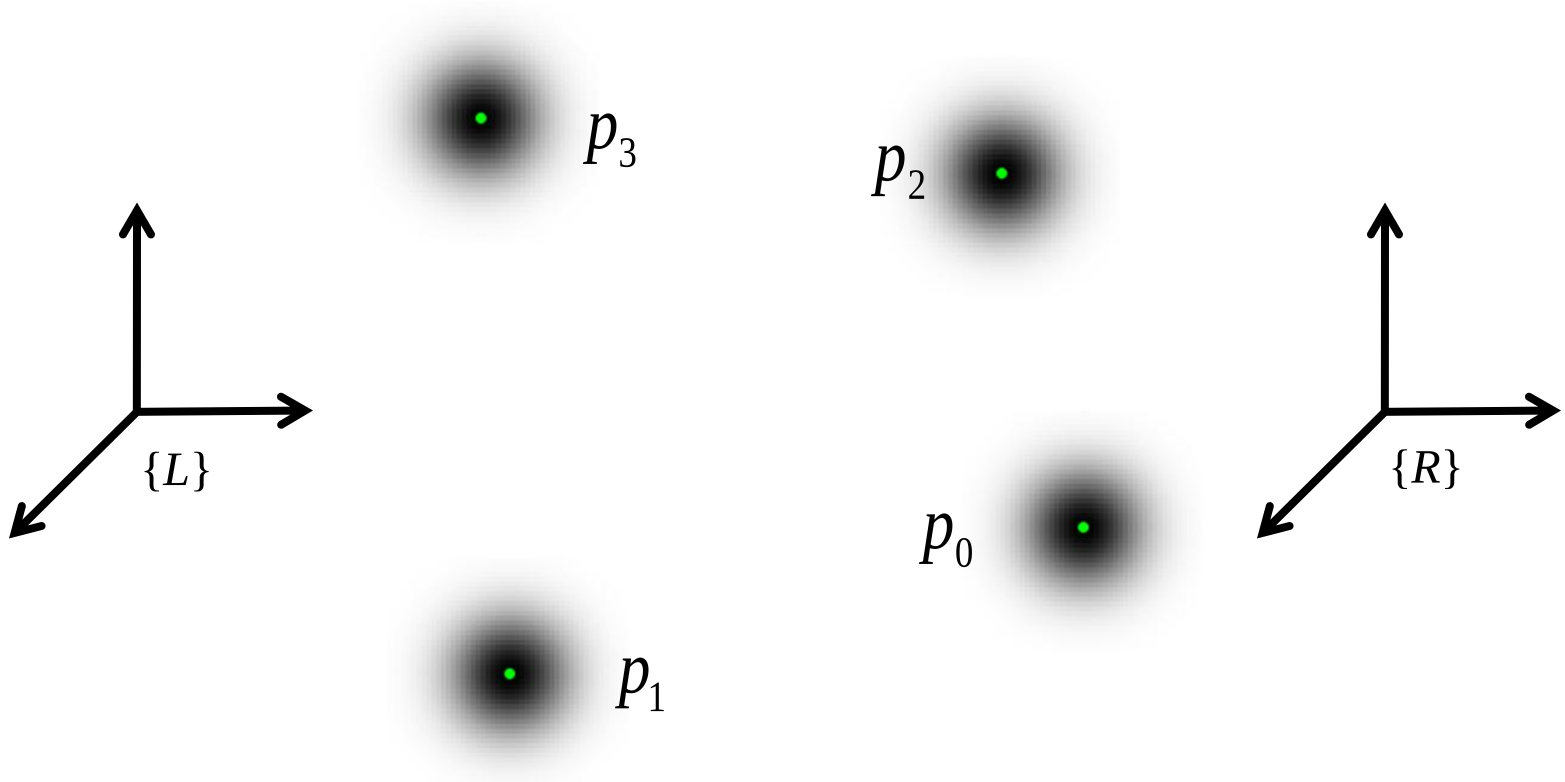
Day 07

Horn's Method and Fiducial Localization Error

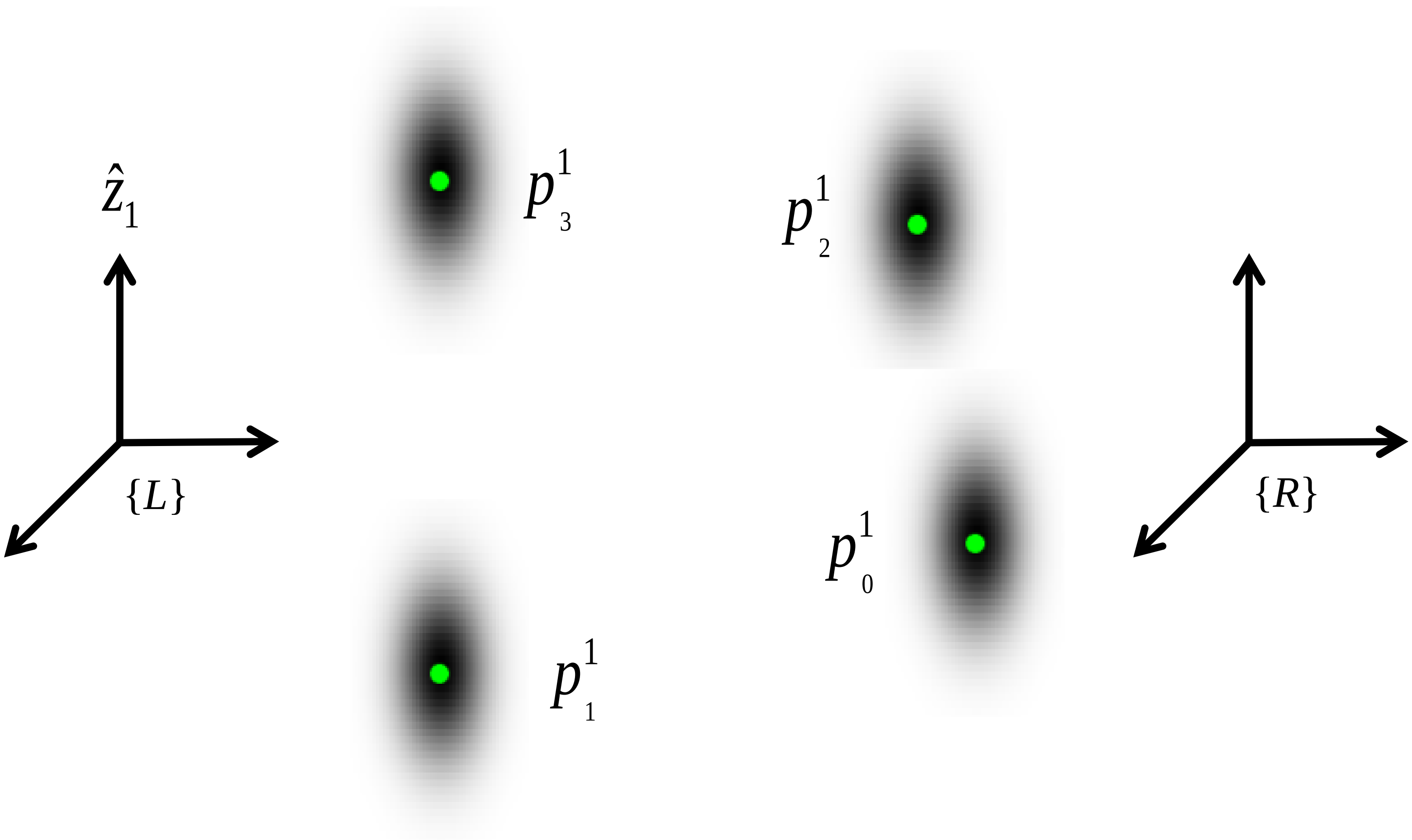
Horn's Method and FLE

- Horn's method (and all other ordinary least-squares methods) is optimal when FLE is identical and isotropic



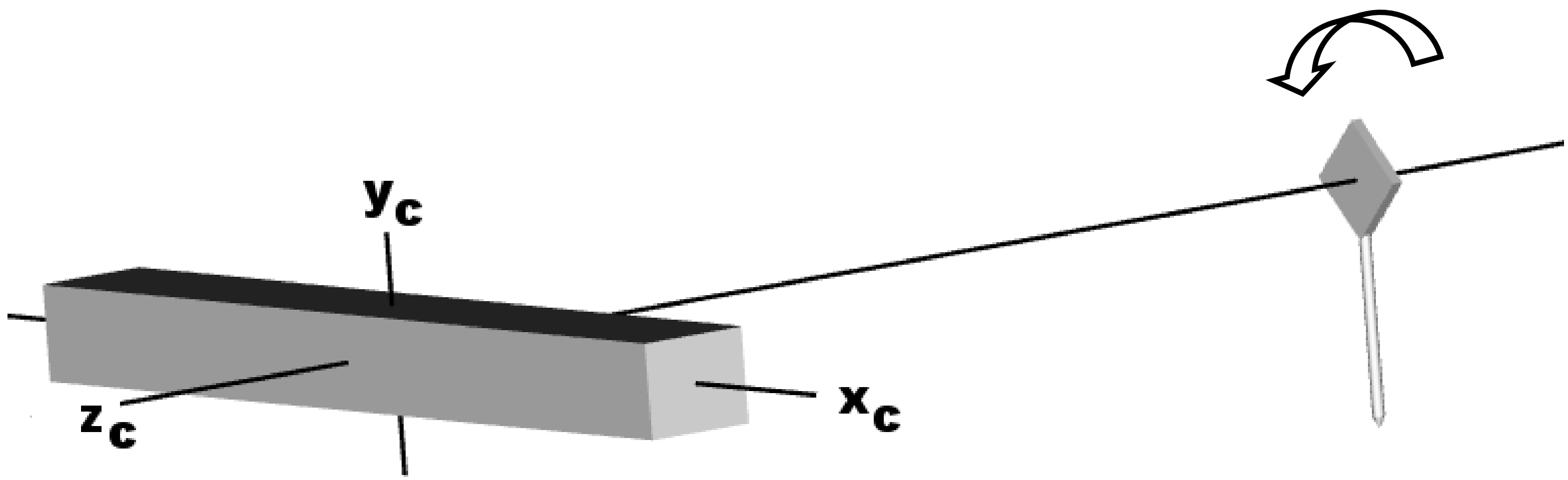
Anisotropic FLE

- in most optical tracking systems, measurement error is largest along the viewing direction



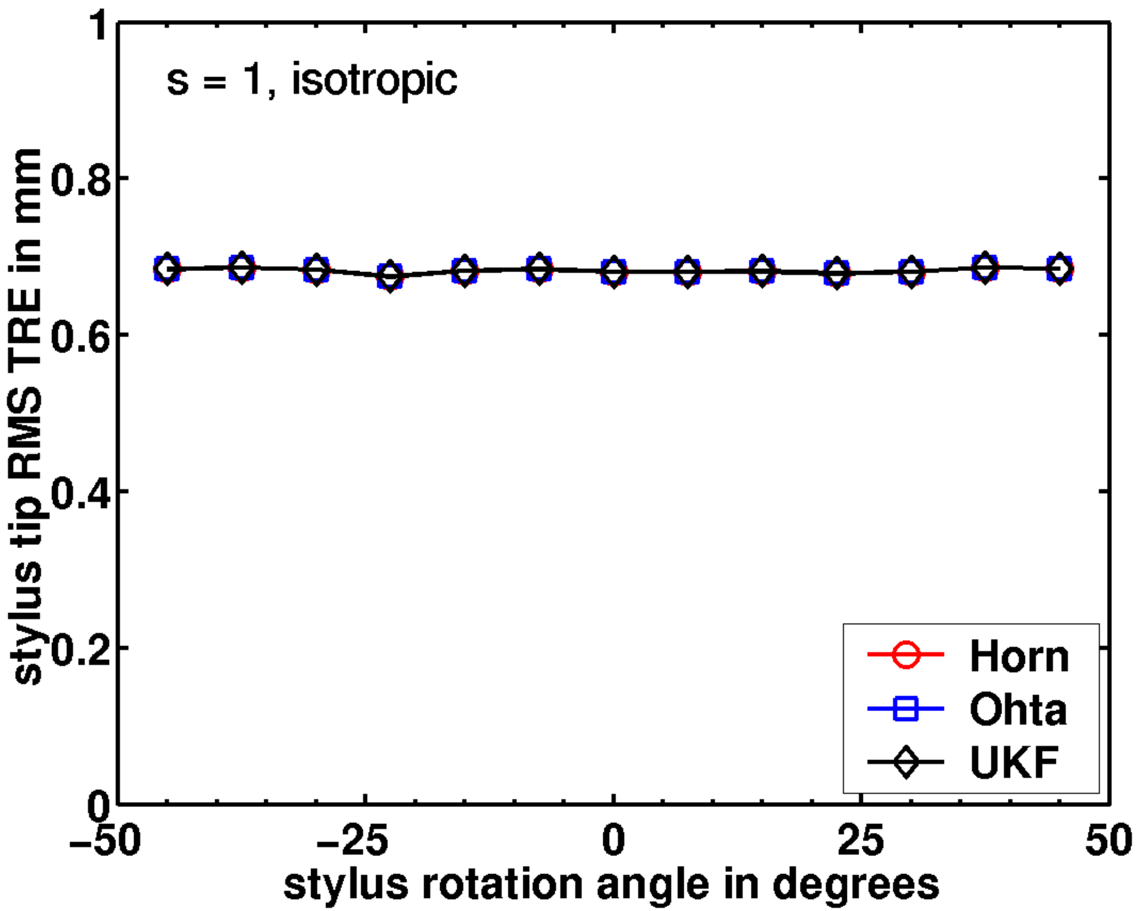
Anisotropic FLE

- what happens if the DRB rotates about x-axis?



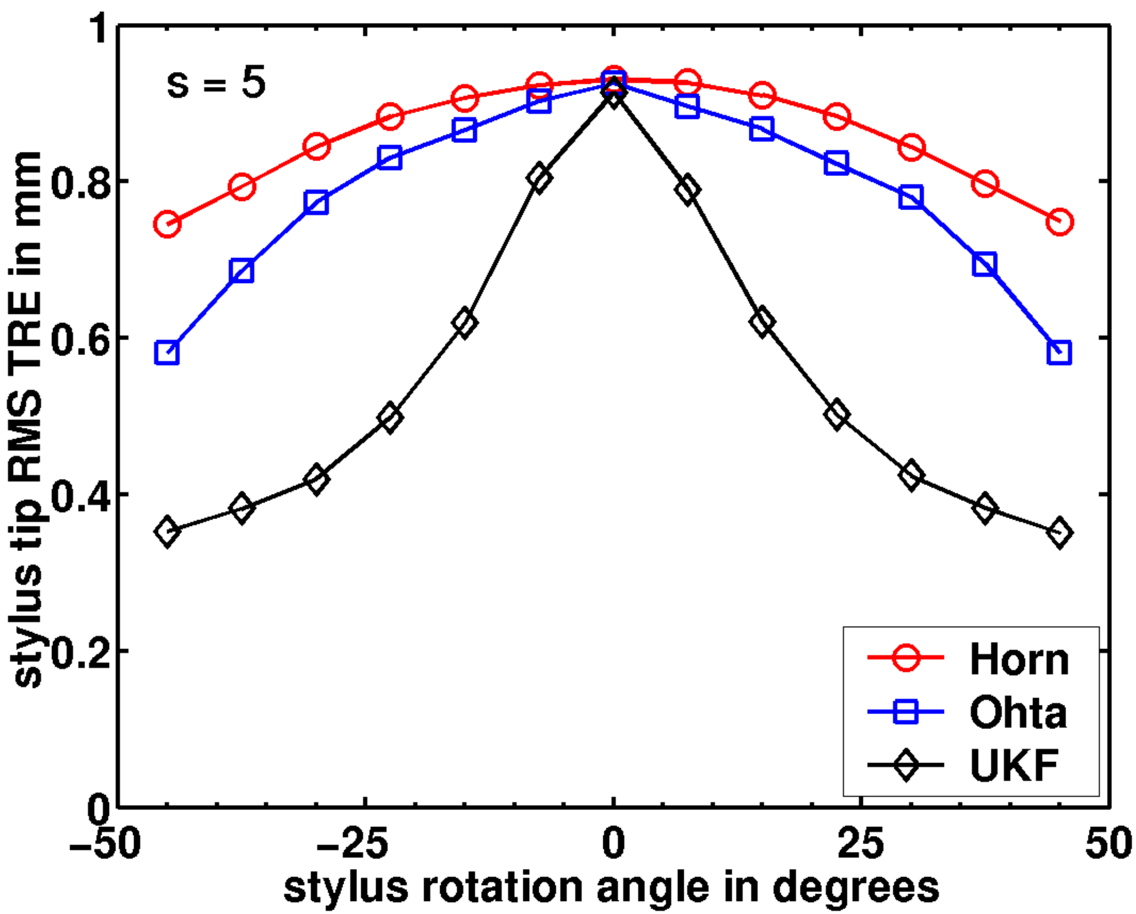
Isotropic FLE

- TRE independent of rotation for isotropic noise



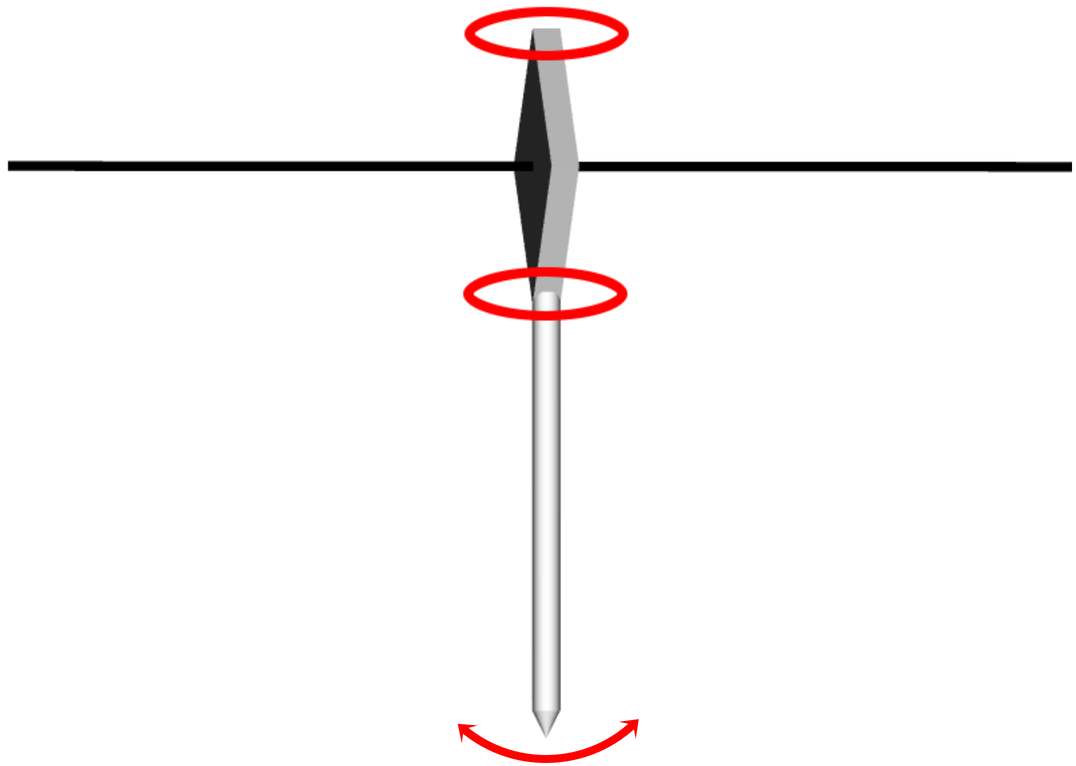
Anisotropic FLE

- TRE strongly dependent of rotation for anisotropic noise



Why the Peak in TRE?

- because rotational component of TRE_{RMS} is maximized when DRB faces the tracking camera



Why the Peak in TRE?

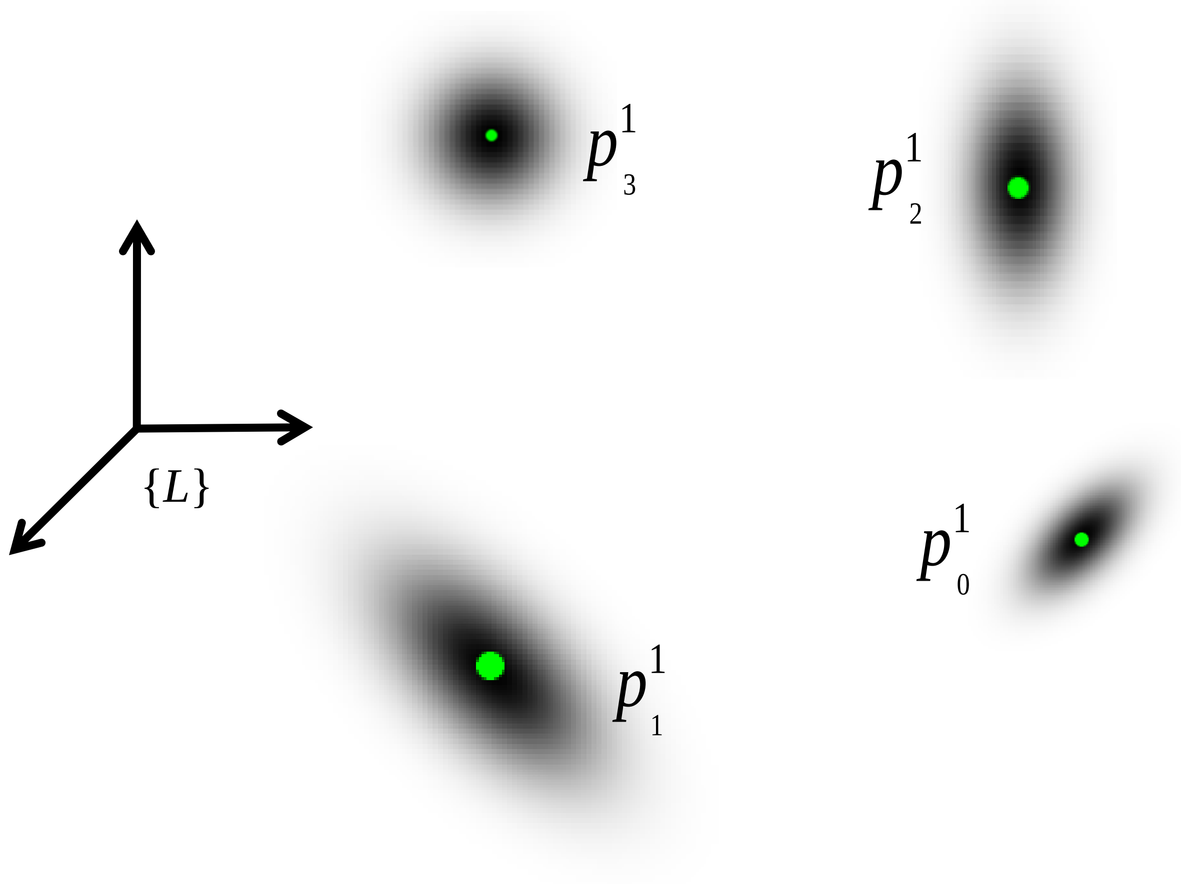
- and minimized when the DRB is perpendicular to the tracking camera

Observed TRE

- paradoxically, this behavior is exactly the opposite of what is observed in practice
  - TRE is typically worse when the DRB is rotated away from the camera

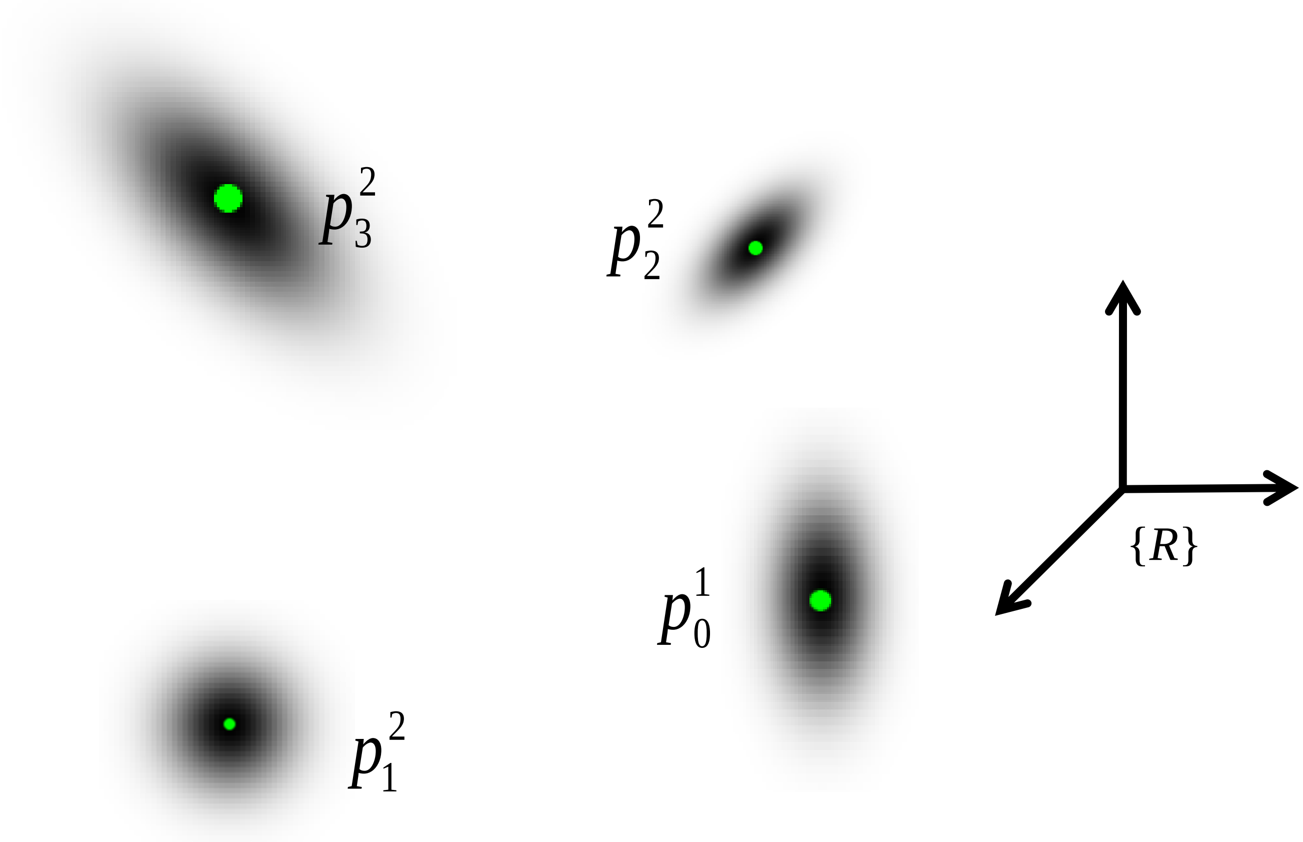
Heteroscedastic FLE

- in reality, noise in each point almost certainly comes from different distributions



Heteroscedastic FLE

- furthermore, both sets of measured points almost certainly contain noise



Heteroscedastic FLE

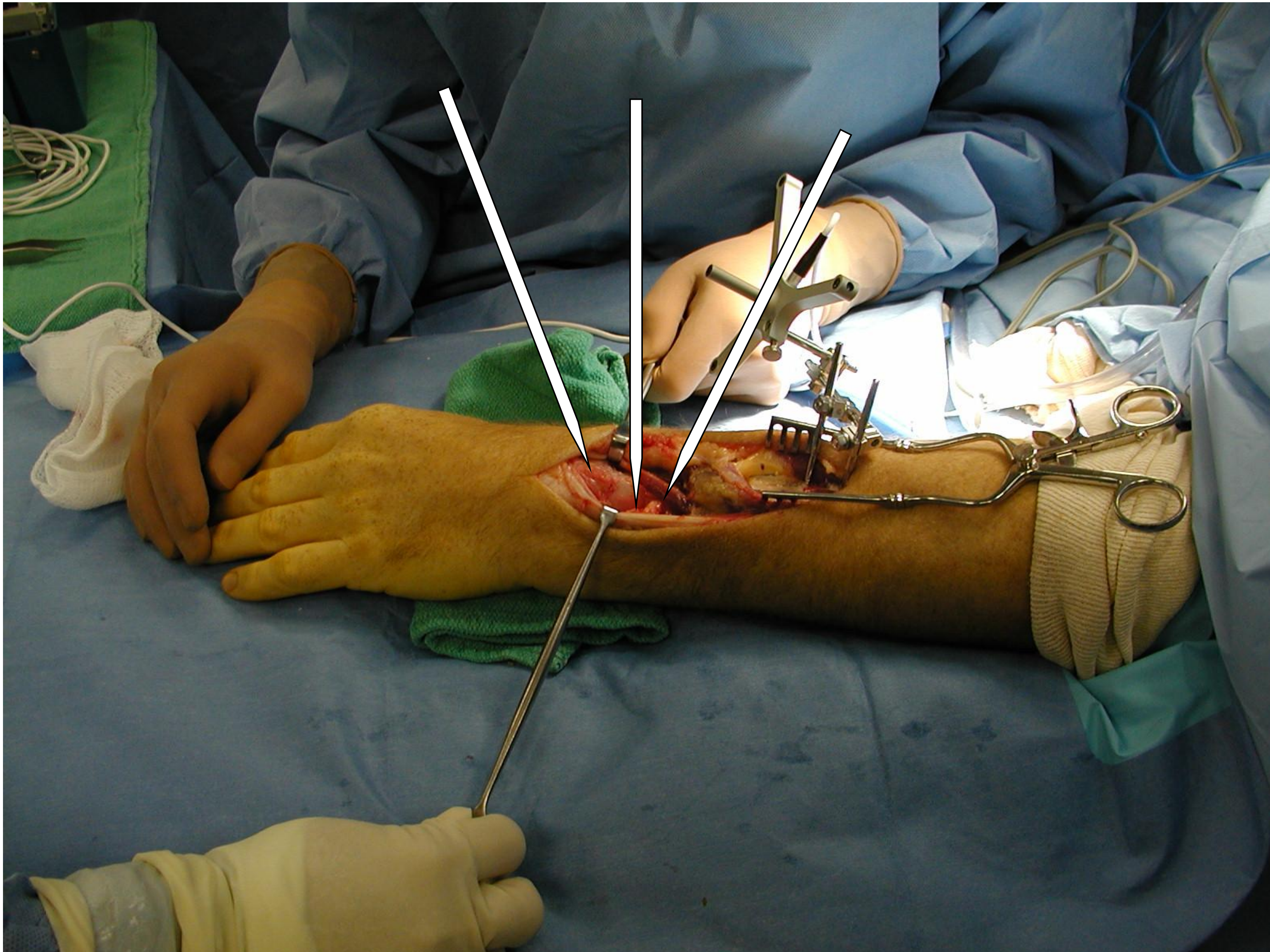
- Estimation of nonlinear errors-in-variables models for computer vision applications. *IEEE Trans. Pattern Anal. Machine Intell. , 28, 1537-1552, 2006.*
- arguably better is Matei's PhD dissertation
  - http://coewww.rutgers.edu/riul/research/theses/abstract/matei_thesis.html
    - ellipse fitting
    - fundamental matrix estimation
    - absolute orientation estimation
    - camera calibration
    - trifocal tensor estimation

HEIV Algorithm for Absolute Orientation

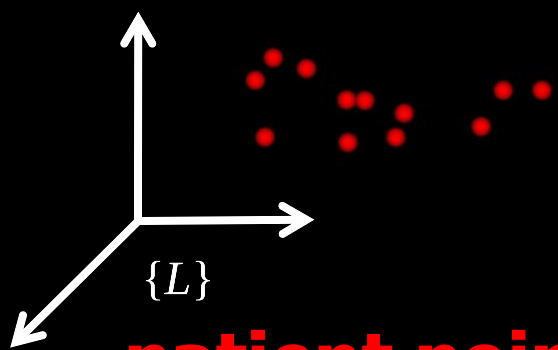
1. Compute the matrices Z_i (one for each registration point) from the original noisy measurements
2. Compute the covariances of the carriers.
3. Use Horn's method to obtain an estimate of the solution.
4. Use the HEIV algorithm to obtain an estimate of the quaternion (encodes rotation) and intercept (encodes the translation)
5. Recover the rotation matrix (from the quaternion) and the translation (from the intercept)

Day 07

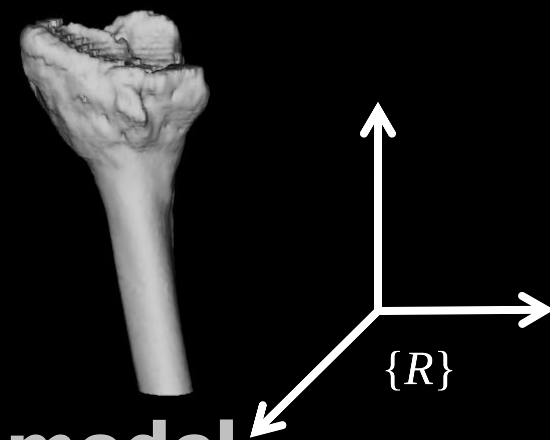
Shape-based Registration



$P_n = \{\mathbf{p}_{i=1..n}\}$



patient points



model

- find rotation and translation that best matches (registers) the points to the surface
- batch process
  - how many points?
  - where are the best points?
  - how do you improve a registration?
  - how good is the registration?

registered

Relationship to Absolute Orientation

- notice that the shape registration problem is similar to the absolute orientation problem
  - the important difference is that we do not know the point correspondences
    - i.e., we do not know which surface point x_i corresponds to registration point p_i
- if we did know the correspondences then the problem is solved; just use any solution to the absolute orientation problem

ICP Algorithm

- iterated closest point algorithm
  - Besl and McKay. IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 14, no. 2, Feb 1992.
  - solves for the rotation and translation that best aligns a set of points to a surface
    - surface can have any representation that allows for a point-to-surface distance computation

ICP Algorithm

1. Find an initial guess for the rotation R and translation d.
2. Apply R and d to P to obtain $Q_0 = \{q_1, \ldots, q_n\}$
3. For k = 1, 2, ...
   1. Find the points on the surface $Y_k = \{y_1, \ldots, y_n\}$ closest to Q_{k-1}.
   2. Use Horn's method to match P to Y_k; this yields the new estimate for R and d.
   3. Apply R and d to P to obtain $Q_k = \{q_1, \ldots, q_n\}$
   4. Compute the mean squared error

$$\delta_k = \frac{1}{n}\sum_{i=1}^{n}\left\| y_i - q_i \right\|^2$$

Until $\delta_{k+1} - \delta_k < \tau$ for some threshold value τ